The tortuous behavior of lightning

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Abstract

The complex branched structure of lightning induce scientists to think that dielectric breakdown is a very complicated phenomena, we will show that this is not true and that simulating the structure of lightning is an easy task, but depends strongly on boundary conditions. In this work we will introduce a new way of understanding the origin of this tortuous path that relies on minimizing the total energy stored in the system.

PACS numbers: 47.54.+r, 61.43.Hv

It is well known that after charging a conductor, the electric charge try to spread out as much as possible on its surface as a consequence of the mutual repulsion between charges of the same sign. When this charged conductor has a sharp end or tip, the electric field just outside this region is bigger than the field outside other points that are far from the tip, and when the electric field exceeds a certain value, the dielectric medium (air for example) will break and a discharge or spark will be initiated at the tip. A similar scenario occurs in thunderstorms[1], where air currents separate negative and positive charges producing regions of high electric fields, and when this field exceeds the value for breaking the air a lightning stroke is produced. The lightning discharge in thunderstorms and sparks between charged conductors, span scales from millimeters to kilometers and evolve following a tortuous path, forming a branched structure that closely resembles a fractal[2].

Negative cloud to ground (CG) flashes account approximately for the 40% of all discharges in natural lightning, there are also: intra-cloud, cloud-to-cloud and cloud-to-air discharges[3, 4]. A typical negative CG flash last for about half a second and can lower an electrical charge of some tens of coulombs. This flash can include three or four current pulses called strokes that last about a millisecond and are separated in time for several tens of milliseconds. These CG lightning discharges are initiated by a downward-moving negative stepped leader, each step having a typical duration of 1 μs, tens of meters in length and pauses between steps of 20 to 50 μs. In its way from cloud to ground the stepped leader produces a typical downward-branched structure, the average velocity of propagation is about 2 $\times 10^5$m/s, the average leader current is between 100 and 1000 A, and the potential difference between the lower portion of the leader and the Earth has a magnitude in excess of 10^7V. As the tip of this negative leader nears ground, the electric field at sharp objects on the ground increases until it exceeds the breakdown strength of air. At that time, one or more upward-moving discharges are initiated from those points, and an attachment with the downward-moving leader occurs some tens of meters above ground. Then the first return-stroke heats the leader channel to a peak temperature near 30,000 K producing thunder, and after several strokes flashing in this channel disappear.

When one is faced with the problem of simulating lightning the first thing to do is to simplify the geometry of the problem. The most used geometries are: a circular conductor (outer electrode) plus a central electrode (see Fig. 1), and an elongated rectangular region with a small electrode in one end and the other end acting as the other electrode. Then an

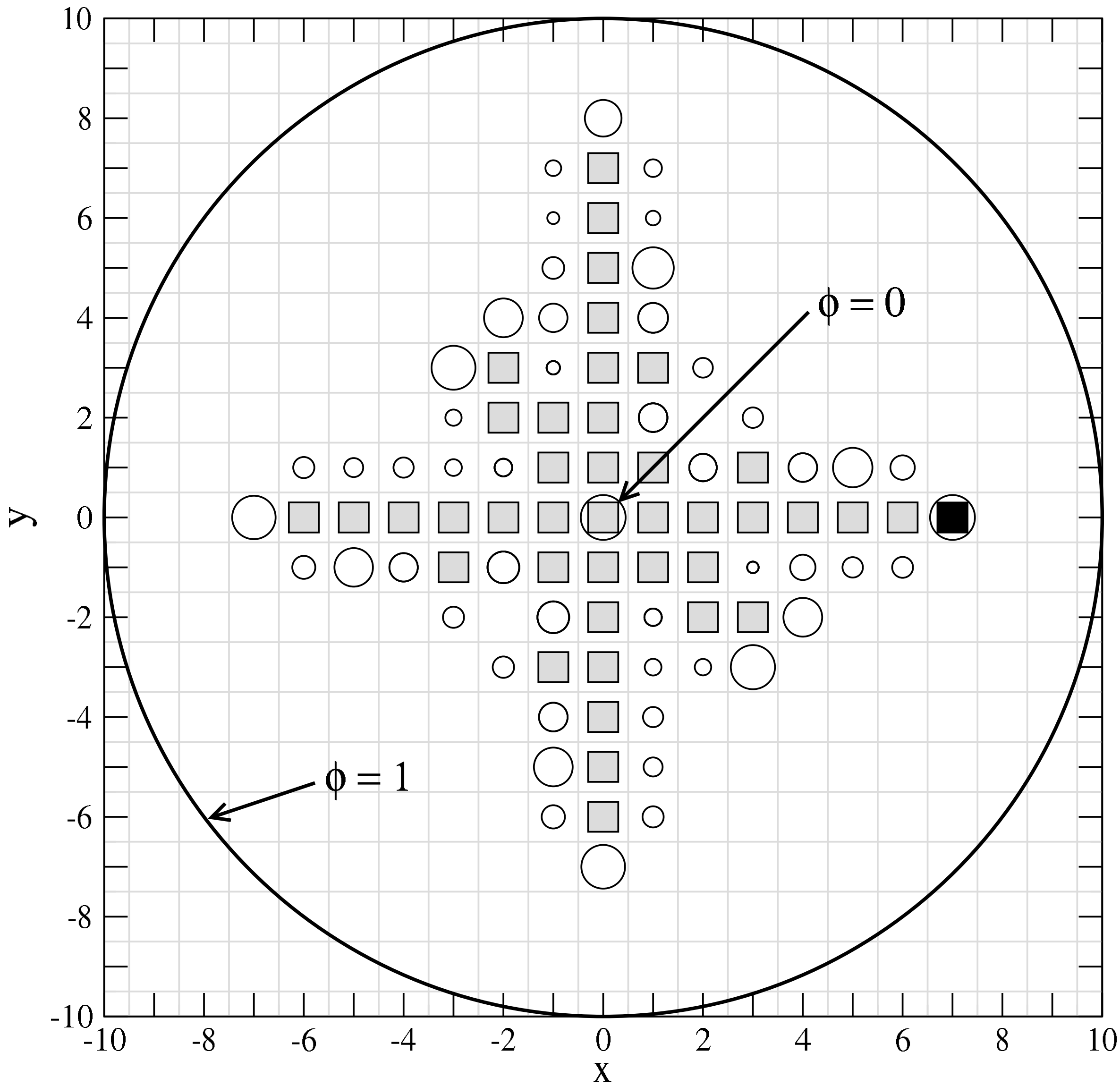

Figure 1: Schematic view of the two-dimensional circular geometry used in our calculations.

electric potential is applied to each electrode, the difference of potential must be big enough to reach the necessary electric field to break the dielectric medium. All conclusions obtained in these two-dimensional geometries can be readily applied to the real 3-dimensional case.

In the theoretical study of the evolution of the discharge channel, the simplest possibility is to expand the inner electrode towards the regions of maximum electric fields. For example, the evolution in the circular geometry proceed in the following way: first, implement boundary conditions for the potential ϕ in the central and outer electrodes; second find the

region of higher electric field (this will be a neighbor of the inner electrode); third, extend the inner electrode as to include the region of high electric field; fourth, calculate again the electric field using the new boundary conditions (the inner electrode has a different form but the potential is maintained constant) and repeat this procedure until the outer electrode is reached. This necessarily gives as a result a straight line between both electrodes as a consequence of the tip effect mentioned previously.

There exists several models in the literature that attempt to find the tortuous structure of lightning, based in procedures similar to the one mentioned in the previous paragraph. The presently accepted model of lightning[5, 6, 7] was developed by Niemeyer, Pietronero and Wiesmann in 1984, and follow these steps but includes a stochastic term that weights a probability that is a function of the value of the local electric field, this model is known as the Dielectric Breakdown Model (DBM) and produces a branched structure whose fractal dimension is similar to the ones obtained experimentally for the same geometry.

In this work we will show that it is possible to obtain a much more realistic branched structure of lightning that follow from a deterministic treatment, that only relies in minimizing the total energy stored in the system and changing locally the dielectric permitivity of the medium (not the geometry of the inner electrode) at each step of iteration.

Before proceeding to explain our model of lightning, we must review some physical concepts that will help us to understand the underlying simple mechanisms acting on the system to produce the branched structure.

The parallel plate capacitor is a very simple electrical device that has all the necessary ingredients to understand the physical concepts[8] under work in our problem at hand. This device consists of two parallel conductive plates (of area A) separated by a short distance (d) and filled with a dielectric material of dielectric permitivity ε. The energy stored in the electric field can be obtained from

$$U = \frac{1}{2} \int \varepsilon (\mathbf{E} \cdot \mathbf{E}) dV. \tag{1}$$

If we disconnect the battery after charging the capacitor, the charge in this device will be held constant, and the energy stored in the electric field (the total energy) will be proportional to d/ε. Physical systems always evolve trying to reduce the total stored energy[9], this implies a force between the plates trying to reduce the distance d, and if you insert a slab of a material having a dielectric permitivity ε' greater than ε, the slab will be pulled into

the capacitor. Now we came to a key point for our model, if instead of maintaining Q constant the potential difference is maintained constant, the energy stored in the electrical field is proportional to ε/d. In experiments at constant Q or V there are forces trying to reduce d and to increase ε. From this we learn that in experiments at constant V, U is not the total energy of the system because the system evolves trying to increase U. There is a missing energy term that comes from a rearrangement of charges in the wires to maintain V constant. When this term is introduced, the total energy is again proportional to d/ε. In short, we can study systems at constant V by letting them to evolve towards regions of higher U (or C).

The electrostatic energy U can be obtained as follows: First, impose the boundary conditions: ϕ=0 in the inner electrode and ϕ=1 in the outer electrode. Second, impose a fixed constant value for the dielectric permitivity ε in all the region between the electrodes. Third, solve the poisson equation

$$\nabla \cdot (\varepsilon \nabla \phi) = -\rho, \tag{2}$$

obtaining the potential ϕ in the region between the electrodes, in our case we do not consider the presence of free charges in this region and we set the charge density ρ to zero. Fourth, obtain the electric field by taking the gradient of this potential, and using Eq. 1, obtain the electrostatic energy.

In the case of cylindrical symmetry, the electric field and the energy U can be obtained analytically. After the first step in the evolution of the discharge channel the system loose this symmetry, and we have to rely to numerical methods for obtaining the energy U.

Now we will study the dielectric breakdown in the circular geometry of Fig. 1. We consider a two-dimensional square lattice, where the central point is the inner electrode and the outer electrode is modeled as a circle. The boundary conditions, ϕ=0 in the inner electrode and ϕ=1 in the outer electrode, are maintained trough all the steps in our simulation. The first step is to assign a fixed value ε to the dielectric permitivity of each lattice point between the electrodes and set the discharge channel to the central point. The second step is to obtain the energies U of the system after changing the dielectric permitivity in one of each neighbor of the discharge channel to a greater value ε', these energies are compared and the neighbor providing the bigger energy value is added to the channel. This last step is repeated until the channel reach the outer electrode.

There are several points worth to mention about our model of lightning: To maintain the

model simple, channel evolution through the diagonals are not permitted. Tip and screening effects are obviously incorporated, but they are not the only essential points in the model. Changing the local dielectric permitivity in not as drastic as changing the geometry of the inner electrode. The local change in permitivity is a key point of our model, providing the mechanism for the system to develop a branched structure in the evolution towards a configuration that minimizes the total stored energy. Our simulated discharge channels for small lattices don't look very realistic, but obtaining better results using bigger lattices, would require to implement sophisticated numerical methods.

Fig. 1 shows a discharge channel for a 20x20 square lattice after 40 iterations. The inner electrode is represented by the central site and the outer electrode is represented by a discretized version of the big circle. For each different configuration, the numerical solution of Eq. 2 was accepted when the numerical residual was less than 10^{-4}, the values for the dielectric permitivity outside the channel was ε=1 and inside the channel was ε'=5. The filled boxes represent the discharge channel (sites where the permitivity is ε'). The circles show all possible sites where the channel can evolve, the diameter of each of these circles represent the value for the energy U of the system in case this site is added to the channel. The circle at the extreme right of the figure, having the black box inside, represent the site giving the biggest contribution to the system energy and is the next step in the evolution of this discharge channel.

The evolution of the discharge channel shown in Fig. 1 shows several important aspects that are not present in other models of lightning: The central site grows at the beginning evolving towards each neighbor, forming a central cross. Then each of the four tips is prolonged forming a bigger central cross (only one of these tips would grow if only the tip effect were at work). This central cross increase size until some branches develop. Although diagonals are not permitted, the channel performs a zigzag evolution to form branches from the central site along the diagonals.

Upward-moving discharges initiated from earth attach with the downward-moving leader in real lightning. We extended our model to consider two initial branches: the main branch coming from the central lattice site and the return branch coming from the outer electrode. Apart from considering the possible evolution of the system by extending the return branch, no other changes were made to the numerical implementation of our model. In Fig. 2 we show the evolution for the same system as the one used for obtaining Fig. 1, but including

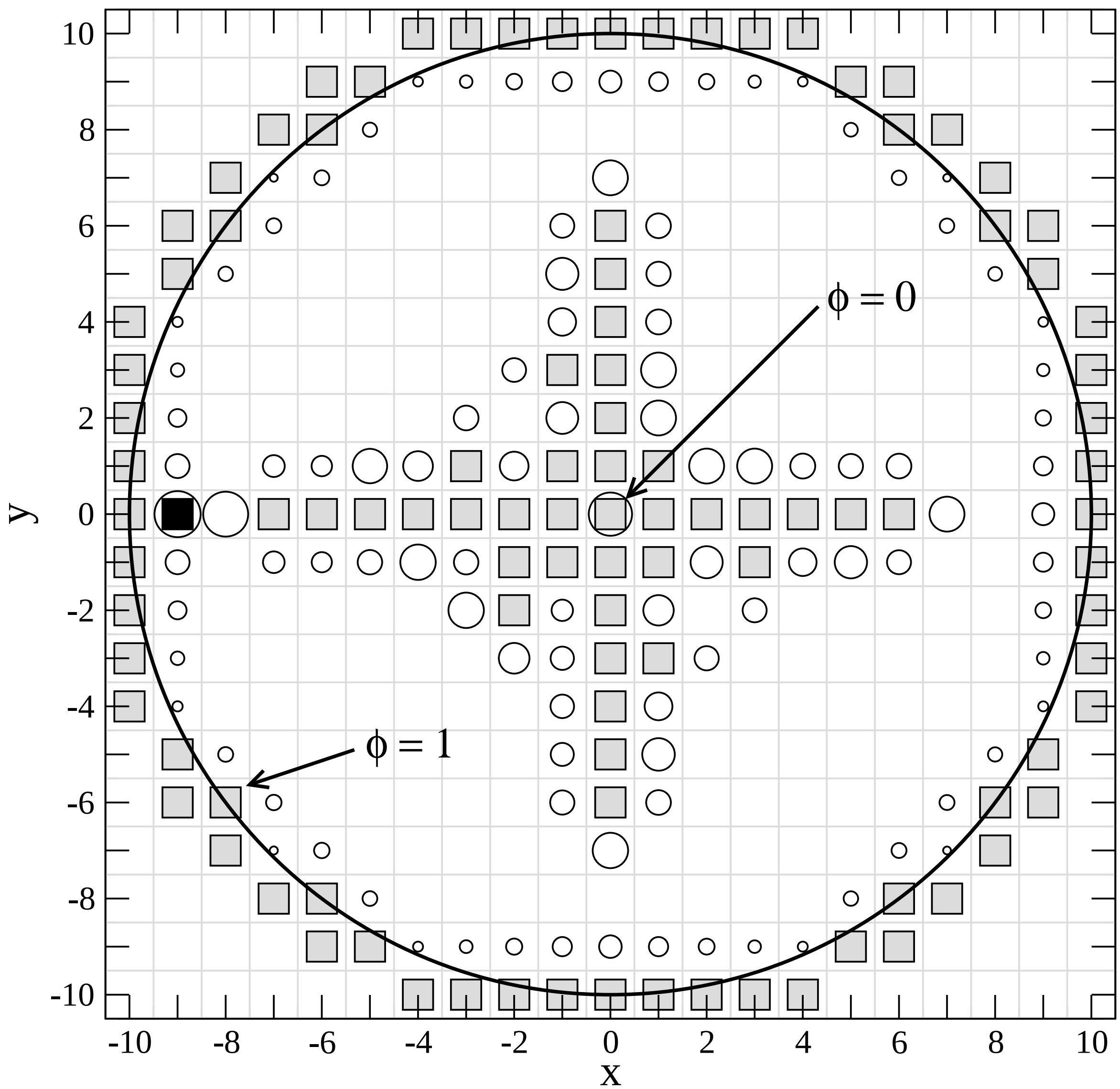

Figure 2: Discharge channel, for a 20 x 20 square lattice, showing the attachment between the return branch (outer gray boxes) and the main branch (inner gray boxes).

the possibility of a return branch and accepting the numerical solution of Eq. 2 when the numerical residual was less than 10^{-2}. The return branch begin to evolve, after 36 steps of evolution of the main branch, by extending the return branch towards the site having the black box inside (at the extreme left of the figure). The site between this point and the main branch is the next step in the evolution, completing the path for this discharge channel from the inner electrode towards the outer electrode. Although the configuration

of the inner part of the discharge channel is the same for Fig. 1 and Fig. 2 (rotated), its evolution after the attachment is different. Obtaining the return branch in our numerical simulations is a really satisfactory finding, as nobody dreamed in the possibility of obtaining this attachment using any of the present models of lightning.

Presently, there is a great scientific interest in the realistic modeling of lightning[10] for: improving models of storms for climate studies; helping in the prevention of aircraft, spacecraft, and other accidents; etc. The numerical implementation of our model is similar (although conceptually different) to others models of lightning, making easy to others groups to check and extend our results. It would be interesting to test if the lightning channel is already formed before any electrical current is transported by the channel, this could be achieved in experiments using optical methods for measuring the differences of refraction index in the system.

The physical mechanism underlying our work is not limited to explain the branched structure of lightning, and we expect that our work will help to understand the origin of branched structures in many other physical systems.

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